Opinion formation models with extreme switches and disorder: Critical behavior and dynamics

Kathakali Biswas [1,2] and Parongama Sen [2]
[1] Department of Physics, Victoria Institution (College), 78B Acharya Prafulla Chandra Road, Kolkata 700009, India
[2] Department of Physics, University of Calcutta, 92 Acharya Prafulla Chandra Road, Kolkata 700009, India

(Received 4 January 2023; accepted 12 April 2023; published 2 May 2023)

In a three-state kinetic exchange opinion formation model, the effect of extreme switches was considered in a recent paper. In the present work, we study the same model with disorder. Here disorder implies that negative interactions may occur with a probability p. In the absence of extreme switches, the known critical point is at $p_c = 1/4$ in the mean-field model. With a nonzero value of q that denotes the probability of such switches, the critical point is found to occur at $p = \frac{1-q}{4}$ where the order parameter vanishes with a universal value of the exponent $\beta = 1/2$. Stability analysis of initially ordered states near the phase boundary reveals the exponential growth (decay) of the order parameter in the ordered (disordered) phase with a timescale diverging with exponent 1. The fully ordered state also relaxes exponentially to its equilibrium value with a similar behavior of the associated timescale. Exactly at the critical points, the order parameter shows a power-law decay with time with exponent 1/2. Although the critical behavior remains mean-field-like, the system behaves more like a two-state model as $q \to 1$. At $q = 1$ the model behaves like a binary voter model with random flipping occurring with probability p.

DOI: 10.1103/PhysRevE.107.054106

I. INTRODUCTION

To address the problem of opinion formation in a society [1–3], several models with three opinion states have been considered recently [4–20]. Typically these opinions are taken as ± 1 and 0, where ± 1 may represent extreme ideologies. In a recent paper [17], using a mean-field kinetic exchange model, the present authors studied the effect of extreme switches of opinion, which is not usually considered in such models. Several interesting results were obtained; in particular, for the maximum probability of such a switch, the model was shown to effectively reduce to a mean-field voter model beyond a transient time. In this paper we extend the previous work by including negative interaction between the agents which acts as a disorder. Such negative interactions have been incorporated in three-state kinetic exchange models previously [10–14] and several properties have been studied in different dimensions. However, the effect of extreme switches and negative interaction both occurring together has not been studied earlier. Since these two features can occur simultaneously in reality, the dynamics of a model incorporating both is worth studying. In the absence of the extreme switches the critical point as well as the critical behavior is known [10–12]. The interest is primarily to see how the critical behavior is affected in the presence of the extreme switches.

In the present two-parameter model, representing the probabilities of negative interaction and extreme switches, in addition to obtaining the phase boundary and behavior of the order parameter, we have studied the dynamical behavior close to the fixed point. The relaxation of the order parameter from a fully ordered state is also studied at and away from criticality. The static critical behavior as well as the dynamical behavior are found to be similar to the mean-field model without extreme switches. However, we find that the nature of the phases in terms of the densities of the three types of opinions is quite different. Especially, the case with maximum extreme switches in the presence of the negative interaction leads to an interesting mapping to a disordered binary model. As a starting point, the mean-field model has been studied where the majority of the results can be obtained analytically. We derive the time derivatives of the three densities of population in terms of the transition rates which are then either solved analytically or numerically. A small-scale simulation is also made particularly to study the finite size scaling behavior of the order parameter.

In Sec. II, the model is described. Results are presented in Sec. III and some further analyses are made in the last section which also includes the concluding remarks.

II. THE MODEL

We have considered a kinetic exchange model for opinion formation with three opinion values $0, \pm 1$. Such states may represent the support for two candidates or parties and a neutral opinion [17,21,22] or three different ideologies where ± 1 represent radically different ones. The opinion of an individual is updated by taking into account her present opinion and an interaction with a randomly chosen individual in the fully connected model. The time evolution of the opinion of the ith individual opinion denoted by $o_i(t)$, when she interacts with the kth individual, chosen randomly, is given by

$$o_i(t+1) = o_i(t) + \mu o_k(t), \tag{1}$$

where μ is interpreted as an interaction parameter, chosen randomly. The opinions are bounded in the sense $|o_i| \leqslant 1$ at all times and therefore o_i is taken as 1 (-1) if it is more (less) than 1 (-1). There is no self-interaction so $i \neq k$ in general. The values of the interaction parameter are taken

